A Mixed Integer Nonlinear Program for Electric Generation Expansion with Energy and Capacity Pricing

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***Abstract*—This paper proposes a new mechanism to give added incentive to invest in new capacities in deregulated electricity markets. An optimization problem to maximize long term social welfare includes binary variables for the building of new facilities, and continuous variables for generation, i.e. the model is a mixed integer nonlinear program. The new mechanism also includes a new approach to calculate capacity prices in addition to the commodity prices: an auxiliary mathematical program calculates the minimum capacity price that is necessary to ensure that all firms investing in new capacities are satisfied with their profit levels.**

***Index Terms*—electricity capacity Price, mixed integer programming, power system economics, power generation investment.**

I. Nomenclature

Indices and Sets

i Index of new sources of energy.
N A subset of new sources of energy.
j Index of old sources of energy.
t Index of time periods.
s Index of demand blocks (base, intermediate and peak).

Variables

$X_{i,t,s}$ Amount of production from new capacities (MW).
$Y_{j,t,s}$ Amount of production from existing capacities (MW).
$Z_{i,t}$ Binary variable, for build (0) or no build (1) decision.
$q^D_{t,s}$ Demand in time period t and demand block s (MWh).
$Cappr_t$ Capacity price in period t ($/MW).

Parameters

$\alpha_{t,s}$ Discounted slope of inverse demand function ($/MWh^2).
$\beta_{t,s}$ Discounted intercept of linear inverse demand function with price axis ($/MWh).
$cD_{t,s}$ Discounted cost of delivery charges in time period t and demand block s ($/MWh).
r Interest rate per time period.
h_s Allocated hours of each demand block in a period (h).
C_{it} Discounted non-fuel variable cost of generator i in t ($/MWh).
f_{it} Discounted fuel cost for generator i in time period t ($/MWh).
K_{it} Discounted capital cost of building generator i in t($).
FC_i Fixed operating cost for generator i ($/MW).
$Price_t$ Discounted price of energy in period t ($/MWh)
XK_i Installed capacity of generator i (MW).
$excap_{j,t}$ Old capacity of type j existing in period t (MW).
$Capf_{i/j}$ Capacity factor for generator i or j (number).
$av_{i/j}$ Availability factor for generator i (number).
rr_t Reserve margin in time period t (number).
$e_{i/j}$ emission coefficient for generator i or j (tons/MWh).
$ECap_t$ Maximum emission in time period t (tons).
$PolicyCap_N$ Maximum amount of production from all generation in subset N (MW).

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II. Introduction

THERE is a lot of concern in energy markets, regarding lack of sufficient private sector investment in new capacities to generate electricity. Although some markets are using mechanisms to reward these investments directly, e.g., by governmental subsidies for renewable sources such as wind or solar, there is not much theory to guide the process of setting the reward level.

The proposed mechanism involves a long term planning model, maximizing the social welfare measured as consumers' plus producers' surplus, by choosing new generation capacities which, along with still existing capacities, can meet demand.

Much previous research in electricity capacity planning has also solved optimization models, usually with continuous variables only, in linear or non-linear programs. However, these approaches can be misleading when capacity additions must either be zero or a large size, e.g., the building of a nuclear reactor or a large wind farm. Therefore, this paper includes binary variables for the building of large new facilities in the optimization problem, i.e. the model becomes a mixed integer linear or nonlinear program.

It is well known that, when binary variables are included in such a model, the resulting commodity prices may give insufficient incentive for private investment in the optimal new capacities. The new mechanism intends to overcome this difficulty with a capacity price in addition to the commodity price by introducing an auxiliary mathematical program which

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calculates the minimum capacity price that is necessary to ensure that all firms investing in new capacities are satisfied with their profit levels.

III. Economics of Electric Generation Expansion

It is usually assumed that free market mechanisms will let investors obtain enough profit, recovering the capital and operating costs of their operation if adequate demand level would be available. If price is always marginal cost of a generator, e.g. a peaker, then it cannot cover its capital cost; however price is sometimes set by demand when the system is at capacity.

During peak hours, base generators like nuclear and coal facilities which have high capital cost and low operating cost, produce electricity through the year, and therefore may be able to make enough revenue to recover their operating cost and a large part of their capital cost (perhaps all); however peaking generators have low capital cost and high operating cost and operate for a small fraction of time; therefore they would be able to recover all their costs only by high market clearing prices occurring during peak periods.

A. Price Mitigation

In order to stabilize electricity markets, system operators set short term market rules and regulations like price caps, forward contracts and subsidies which can sometimes create market disequilibrium. For instance, price caps limit electricity prices to levels below market clearing prices in peak periods, when demand is high. Although generators may make positive short-run operating profits (Revenue minus variable cost), to make adequate profit on the investment in generation, they (especially peak generators) rely on very high prices (peak prices) that happen occasionally and therefore a price cap can have a negative impact on their incentive to invest in new capacity.

Some advocate abolishing price caps as a solution, but even without a price cap, there can still be a problem of insufficient revenue, due to "lumpiness" of investment in new electric capacity [1]. Therefore, the models of this paper do not include price caps; they focus on the lumpiness of investment and suggest mechanisms to overcome this problem by introducing two part pricing or capacity payments.

B. Capacity Payments

Capacity payments are incentive mechanisms to electricity producers to recover what is called "missing money" [1]. Although capacity payments may induce new generation, the regulator should be careful not to put a huge burden on consumers by very high electricity prices. Furthermore, in countries with imperfect electricity markets, in which some generators have excessive market power, this incentive mechanism might be abused; since those generators can easily exaggerate the value of lost load (VOLL) and therefore increase the amount of capacity payment [2]. Due to this problem, capacity payments in England market went so high that they were 20% of total payments to generators [3].

IV. Pricing Integer Activities

A model of a market with some discrete activities and some continuous activities is a mixed integer program (MIP). Prices are often extracted as dual variables when all variables are continuous, but a MIP model presents special difficulties.

The classic work of pricing integer variables by interpreting dual variables in MIP programs goes back to Gomory and Baumol [4], who looked into dual prices and their relationship with the marginal cost of adding indivisible resources. They introduced a cutting plane methodology (adding new constraints) in order to find a solution to a MIP and used the dual variables of these constraints to price the cost of integer activities; however these extra prices are related to the choice of additional constraints and they do not result in unique answers.

Furthermore, Williams extended Gomory's and Baumol's work by examining the mathematical and economic properties of LP duality and relating them to integer programming dualities [5]. Proving that the dual program, proposed by Gomory and Baumol, doesn't provide optimality, he introduced a more complicated dual problem, satisfying optimality; however it doesn't satisfy complementarity conditions.

In order to resolve missing money problem, in electricity markets, several researchers proposed methodologies to calculate additional capacity payments while maintaining market equilibrium. Scarf raised the problem of indivisibility and equilibrium prices by suggesting that, many activities involve non-convexities or indivisibilities, such as building a new generator, and therefore it makes it difficult to introduce equilibrium prices in such activities [6].

O'Neill et al. proposed a new two part pricing scheme, promising market equilibrium in markets with non-convexities [7]. In this proposal, they introduce an auxiliary linear program, with additional constraints in which binary variables are set at their optimal values, derived from the MILP program. Interpreted in the context of electricity investments, the dual variables of these additional constraints are used as capacity payments, which along with commodity prices provide enough incentive for new investors to invest in electricity markets.

Finally, Fuller proposed a general definition of equilibrium, to price both continuous and binary variables in a more general form of models than social welfare maximization, to include nonintegrable models as well [8]. The ideas were applied to the short run unit commitment problem in [9], in which the binary variables represent the on/off status of generators. An efficient way to calculate commodity prices and binary related prices (as payments in proportion to capacities) was derived by [9] as a modification of the method of [7]. The capacity prices are non-negative and non-discriminatory.

V. Mathematical Models

The first model is a mixed integer non-linear program (MINLP) which chooses new generation to maximize the present worth of consumers' plus producers' surplus (social welfare). It could be executed by either a market operator or a regulator to identify the near optimal number of permits which could be given to potential investors, the starting time of the developments and also the amount of energy that could be produced by new and old capacities in order to achieve long term energy supply goals and targets.

A social welfare maximization model is preferred over other mathematical models such as producer's cost minimization or profit maximization programs, since it reimburses new producers by not putting a huge burden on consumers and therefore maintains the market equilibrium. It is important for a regulator to be confident that generators can recover the cost of new investments, and on the other hand it is vital to support consumers by maintaining the energy price at a reasonable level. A social welfare maximization model addresses both of these critical planning issues.

A. Formulation of MINLP

Figure 1 outlines the general formulation for a social welfare maximization model with respect to long term capacity planning. Since the proposed model is a long term planning program, constraints which are related to short-term operational issues such as ramping time and minimum up and down time are not used. For simplicity, issues related to the transmission system are ignored in the model. Relevant constraints for this model are supply, capacity and policy limitations.

$$
\boxed{\begin{array}{l}
Max \quad Social\ Welfare \\
\\
Subject\ to: \\
\qquad Supply\ Constraints \\
\qquad Capacity\ Constraints \\
\qquad Binary\ Constraints \\
\qquad Policy\ Constraints
\end{array}}
$$

Figure 1: MINLP Model

Social Welfare Objective Function

In Figure 1, social welfare is the total discounted value to consumers (areas underneath linear demand curves), minus the discounted costs incurred by producers, as given by the expressions (1) to (3). Note that all monetary value parameters are in present worth, except for FC_i.

$$\sum_t \sum_s \{\beta_{t,s} * q^D_{t,s} + {}^1\!/_2 * \alpha_{t,s} * {q^D_{t,s}}^2 - cD_{t,s} * q^D_{t,s}\} \tag{1}$$

$$\begin{aligned} -\sum_i \sum_t \sum_s \{ & C_{i,t} * X_{i,t,s} * h_s + f_{i,t} * X_{i,t,s} * h_s \\ & + K_{i,t} * Z_{i,t} + FC_i * Z_{i,t} * XK_i * \sum_{t' \geq t} 1/(1+r)^{t'}\} \end{aligned} \tag{2}$$

$$-\sum_j \sum_t \sum_s \{C_{j,t} * Y_{j,t,s} * h_s + f_{j,t} * Y_{j,t,s} * h_s\} \tag{3}$$

The first part of the objective function (1) is the discounted area underneath the electricity demand curve minus the delivery charges, paid by consumers for each period t and demand block s, and the next part (2) gives the discounted cost of new generation, including non-fuel variable cost, fuel cost, capital cost and fixed operating cost. In this expression, fixed costs occur annually after the construction of the plant. The last part of this objective function (3) takes care of non-fuel variable cost and fuel cost associated with existing capacities. However since fixed operating costs for existing generators are already known through the time horizon of the plan and they don't have any impact on the outcome of the model, we don't consider them in (3). In expressions (1) to (3) parameters cD_{ts}, C_{it}, f_{it}, K_{it}, C_{jt} and f_{jt} are already discounted, and therefore no explicit discount factor is needed in these terms.

Supply Constraints:

Constraint (4) forces the amount of supply by new and old generators to be greater than or equal to demand, in each time period, and each demand block. As already mentioned, demand is a price responsive variable. The produced electricity from new generators ($X_{i,t,s}$) and existing generators ($Y_{j,t,s}$) are in MW, but the demand of $q^D_{t,s}$ is in MWh; therefore the number of hours in each demand block, h_s reconciles the units.

$$\sum_i X_{i,t,s} * h_s + \sum_j Y_{j,t,s} * h_s \geq q^D_{t,s} \quad \forall t, s \tag{4}$$

Capacity Constraints:

Constraints (5) and (6) restrict the amount of production to be no more than the effective potential capacity of each generator, which depends on its capacity and availability factors.

$$X_{i,t,s} \leq XK_i * Capf_i * av_i * \sum_{t' \leq t} Z_{i,t'} \quad \forall\, i, t\ and\ s \tag{5}$$

$$Y_{j,t,s} \leq excap_{j,t} * Capf_j * av_j \qquad \forall\, j, t\ and\ s \tag{6}$$

As shown in (5), the amount of production from a new generator is zero if the generator is not built until that time period. For existing generators, there is no variable for a decision to build; in (6), the parameter $excap_{jt}$ forecasts the capacity of presently existing generation that is expected to be still available in period t.

Binary Constraints:

Constraint (7) limits the model to build each generator no more than once during the time horizon of the plan.

$$\sum_t Z_{i,t} \leq 1 \qquad \forall\, i \tag{7}$$

Policy Constraints:

These types of constraints are different, depending on countries' regulations, policies and long term plans that affect their supply mix. Some constraints such as reserve capacity (8), emission limit (9) and generation limit (10) play an important role in various countries' long term plans. The following equations illustrate these types of constraints:

$$\sum_i X_{i,t,peak} + \sum_j Y_{j,t,peak} + q^D_{t,peak} * rr_t/h_{peak} \leq \sum_i XK_i\, Capf_i av_i \sum_{t' \leq t} Z_{i,t'} + \sum_j XK_j\, Capf_j av_j) \quad \forall t \tag{8}$$

$$\sum_{i,s} e_i * X_{i,t,s} * h_s + \sum_{j,s} e_j * Y_{j,t,s} * h_s \leq ECap_t \quad \forall t \tag{9}$$

$$\sum_{i \in N} X_{i,t,s} \leq PolicyCap_N \qquad \forall\, t, s \tag{10}$$

Equation (8) requires available effective capacity in each time period to exceed or equal the amount of peak production from new and existing capacities plus a reserve requirement given as a fraction of peak power demand. If the model includes the possibility of much intermittent renewable energy, then a second reserve requirement term would be proportional to the expected amount of production from intermittent sources.

Equation (9) limits the amount of emission (e.g., greenhouse gas) caused by different types of electricity generators during each period of the plan. Depending on the value of the parameter $ECap_t$, the constraint may force the generation from coal or gas facilities to decrease and instead wind, hydro and solar capacities to increase.

Equation (10) puts a limit on new generation by a defined subset of generation *N* (e.g., nuclear) for each time period and demand block.

B. *Auxiliary Non-linear program for Pricing*

By solving the MINLP of Figure 1 as detailed in (1) to (10), the time of construction, amount of production and electricity demand in each time period will be identified. In models with only continuous variables, the dual of supply constraint (4) is normally interpreted as the market clearing price. However, in models that include discrete variables, such as the model of Figure 1, it is possible for an investor to have negative profit, which is one form of disequilibrium that is discussed by O'Neill et al. [7]. In order to identify capacity payments or reimbursement for binary activities such as building new capacities, O'Neill et al. suggest using an auxiliary NLP problem in which the binary variables become continuous, but they are fixed at their optimal values; applied to the model of Figure 1, the auxiliary NLP is illustrated in Figure 2. The dual variables associated with the new equality constraints $\gamma^z_{i,t}$, together with the dual variables of the market clearing constraints (4), are proven to be equilibrium capacity payments for each new facility in each time period [7]. With these capacity payments and the market clearing prices, each producer is content to construct new capacity and operate its units as indicated by the optimal solution of the MINLP of Figure 1.

In equations (11), $Z^*_{i,t}$ are the binary variables in Figure 1, which are fixed at their optimal values in Figure 2, and therefore make it an NLP model with just continuous variables. The objective function and all the constraints are exactly as described in the previous section in (1) to (9). The only differences are the conversion of $Z_{i,t}$ from binary to continuous, the inclusion of equality constraints (11), and elimination of equations (7) and (8) due to redundancy.

$$\max_{(q^D_{t,s},\, X_{i,t,s},\, Y_{j,t,s},\, W'_{j,t},\, Z'_{i,t})} \; Social\ Welfare$$

Subject to :

(4), (5), (6), (9)

$$Z'_{i,t} = Z^*_{i,t} \qquad \forall\, i, t \qquad (\gamma^z_{i,t}) \tag{11}$$

$$q^D_{t,s}, X_{i,t,s}, Y_{j,t,s}, W'_{j,t}, Z'_{i,t} \geq 0 \tag{12}$$

Figure 2: NLP Model Based on O'Neill et al. [7]

C. *Auxiliary linear program for Pricing*

Figure 2 suggests discriminatory capacity payments to each new facility in each time period. However in practice it is easier to implement a non-discriminatory capacity price per unit of a generator's available capacity in each time period. Another issue with "O'Neill payments" is that sometimes the dual variables of the equality constraints (11) are negative, meaning that generators should pay for the privilege to construct a new facility, which would be extremely impractical in real world contracts.

In order to overcome the problems with the O'Neill capacity payment, we suggest the following LP minimization formulation, which introduces non-discriminatory capacity prices, while avoiding negative profits for generators. In the LP model of Figure 3, the only decision variables are the capacity prices in different time periods $Cappr_t$, which should be chosen to minimize the present worth of total capacity payments to reduce the burden on consumers and hence increase the consumer's welfare. Furthermore, values of binary variables and amounts of generation have been fixed at their near optimal values, calculated by a MINLP solver applied to the model of Figure 1. On the other hand, discounted market clearing prices $Price_{t,s}$ have been derived from duals of supply constraints (4) in the NLP of Figure 2.

The LP of Figure 3 minimizes the present worth of all capacity payments, assuming that $Cappr_t$ is paid for each MW of capacity that exists in period *t*. Recall that facility *i* exists in period *t* if $\sum_{t' \leq t} Z^*_{i,t'} = 1$. Constraint (14) forces the model to calculate capacity payments such that no loss happens to new generators.

Although it is important to provide positive profit for producers, it should not impact consumers' welfare, such that they have negative surplus. Constraint (15) takes care of this problem by forcing the model to have positive welfare for consumers while providing a reasonable amount of profit for producers. It is assumed that capacity payments are being made directly by consumers. In this formulation, capacity payments haven't been considered for existing capacities, since the goal of the model is to provide incentives for new investors.

Also, in order to force the model to generate zero capacity prices during the time periods that no generator is built, constraint (16) has been added to Figure 3. This constraint equates capacity payments to zero in time periods when there is no new construction; the large number M makes the constraint nonbinding when there is new construction. It expresses the idea of basic economic theory that if there is no need for new capacity in a time period, then there is excess supply of capacity, and the price of capacity should be zero.

$$Min \sum_t \sum_s \frac{Cappr_t * XK_i * Capf_i * av_i * \sum_{t'\le t} Z^*{}_{i,t'}}{(1+r)^t} \tag{13}$$

$(Cappr_t)$
Subject to :

$$\sum_{t,s}\{Cappr_t \frac{\sum_{t'\le t} Z^*{}_{i,t'} * av_i * capf_i * XK_i}{(1+r)^t} + Price_t * X^*{}_{i,t,s}\} \geq \sum_t \sum_s \{(C_{i,t} * X^*_{i,t,s} * h_s + f_{i,t} * X^*_{i,t,s} * h_s + K_{i,t} * Z^*_{i,t} + FC_i * Z^*_{i,t} * XK_i * \sum_{t'\ge t} 1/(1+r)^{t'}) \quad \forall i \tag{14}$$

$$\sum_s \{\alpha_{t,s} q^D_{t,s} + .5\beta_{t,s} {q^D_{t,s}}^2 - Price_{t,s} q^D_{t,s} - \sum_i \{Cappr_t \, XK_i Capf_i av_i \sum_{t'\le t} Z^*{}_{i,t'}\} \geq 0 \;\; \forall t \tag{15}$$

$$Cappr_t \le M * \sum_i Z^*{}_{i,t} \qquad \forall\, t \tag{16}$$

$$Cappr_t \geq 0 \qquad \forall t \tag{17}$$

Figure 3: Proposed LP Model

VI. Numerical Example

This section calibrates the proposed mathematical models based on Ontario's Integrated Power System Plan (IPSP) data [11], and illustrates the O'Neill payments and the proposed capacity prices for electric generation investments in the next 20 years.

Data used in this example replicate IPSP's supply resources and cost estimates; however the inverse demand function parameters are based on the available demand elasticity estimates, together with the IPSP forecasts of demand quantities and prices. Therefore in the model, demand is price responsive, while in IPSP, demand is forecasted based on 1.1% average annual growth rate [11]. Another major difference between this numerical example and IPSP is that IPSP doesn't consider different prices for different demand blocks such as peak, intermediate and base, while the proposed model has the capability of forecasting prices in various demand blocks, and therefore makes it possible to forecast demand during each demand block.

According to IPSP, reserve margin (8) and generation limit (10) policy constraints have been used in the model. The generation limits are 14000 MW for nuclear, 10200 MW for gas, 450 MW for bio fuel, 4039 MW for wind and 4921 MW for hydro.

The mathematical models have been programmed in GAMS (General Algebraic Modeling System), with the SBB-CONOPT solver for the MINLP and CONOPT solver for the NLP and CPLEX solver for the LP. For this example, the MINLP contains 6,069 binary variables, 18,271 continuous variables, and 18,786 constraints; the NLP has 24,235 variables (all continuous) and 24,540 constraints; and the LP has 22 variables and 282 constraints. The computational effort is reasonable: all three models are solved within a few minutes.

Figure 4 shows some of the new generations selected by the MINLP from 280 possible new investments. Some peak or intermediate generators like Gas and Bio will have negative or near zero profit, if they are compensated by only market clearing prices, and therefore they will not have any incentive to make the investments that are indicated by the solution of the MINLP.

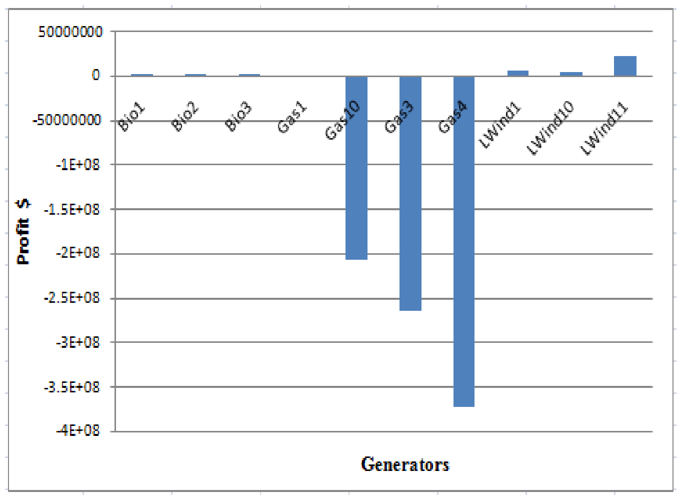

Figure 4: Net Profit with no Capacity Payment

In order to give sufficient incentives for new investments, O'Neill capacity payments of $\gamma^z_{i.t}$, which are dependent on type of generation and time of construction, have been proposed. As already described, these capacity payments are dual variables of constraint (11). Although there are dual values for each generator for time periods without any construction, those dual values are not economically important because they represent "prices" that would be multiplied by zero valued variables, for zero revenue, and therefore they haven't been considered in capacity payment calculations. They simply show that if the generators are offered those dual values, they still won't have enough incentive to construct new capacities in those periods. Figure 5 shows the amount of profit that the generators of Figure 4 will make after receiving additional O'Neill payments.

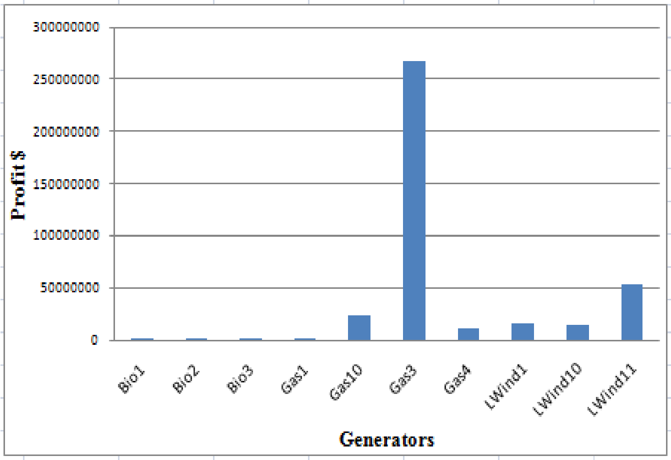

Figure 5 : Profit with Commodity Prices and O'Neill Capacity Payments

As illustrated in the above figures, those generators that were making negative profit receive the most capacity payments, such that they can make non-negative profit at the end of the plan's time horizon. Other generators also receive additional payments; however, it is difficult to give an economic rationale for these payments -- they are simply the mathematical result of the O'Neill method.

In order to overcome problems with the O'Neill approach, the proposed non-discriminatory capacity prices are shown in Figure 6. Only the positive prices are shown – for years 9, 10, 11 and 12. All other periods had zero capacity prices.

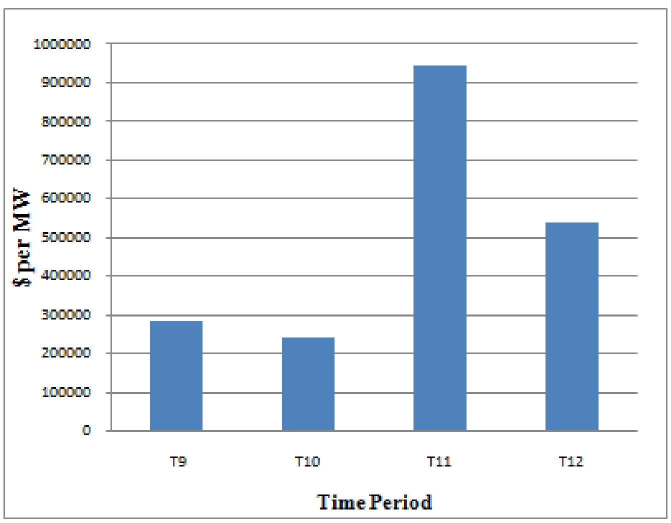

Figure 6 : Proposed Capacity Prices

The proposed capacity prices result in positive profits for all generators, which are shown in Figure 7 for the selected producers.

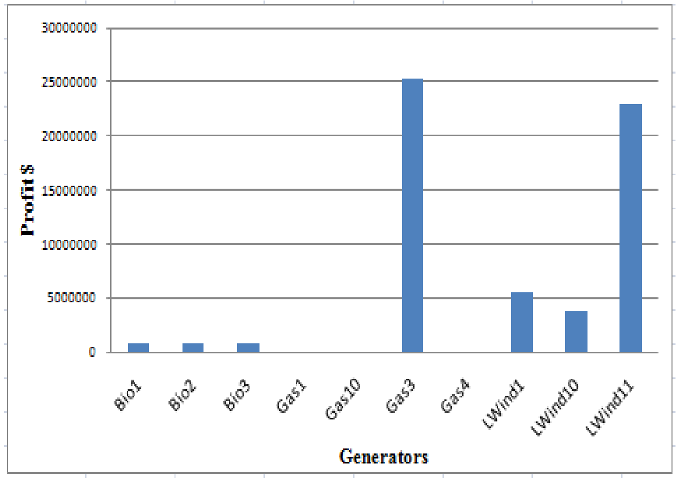

Figure 7: Profit with Proposed Capacity Prices

VII. CONCLUSION

This paper introduces an MINLP social welfare maximization program for a long term capacity expansion model, including price responsive load in different demand blocks, used to calculate the amount of generation from new and existing facilities, time of construction and electricity demand in each period. Furthermore, it examines the adequacy of commodity prices which result from the duals of market clearing constraints in order to see whether they produce satisfactory incentives for investment.

Since these prices are sometimes insufficient for producers to recover their capital cost, the paper introduces a two-part pricing scheme, which pays new generators capacity payments in addition to commodity prices.

In order to calculate capacity payments, the paper examines the approach of O'Neill et al. [7], which includes an auxiliary non-linear program, with additional constraints in which binary variables are set at their optimal values. The dual variables of these constraints are used as capacity payments, promising that these extra payments provide market equilibrium.

However, due to criticisms of the O'Neill approach, this paper introduces an auxiliary LP program, calculating non-discriminatory capacity prices by minimizing total present worth of all capacity payments while guaranteeing that the new generators have positive profit and also consumers' surplus is greater than zero.

Furthermore, the paper illustrates the suggested models in the context of Ontario's Integrated Power System Plan.

This paper can be extended in future by incorporating technologies with continuous capacity additions like small scale distributed generation

Moreover, the proposed models can be used in calculating other policy instruments like Feed-In-Tariff. Knowing the cost of new generators, the models can estimate a reasonable amount for premiums, in order to encourage investments in electricity markets.

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